Structural Estimation of a Gravity Model of Trade with the Constant-Difference-of-Elasticities Preferences

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Gravity model is a well-known quantitative trade model that analyzes impacts of size of bilateral partners and their distances on trade patterns.

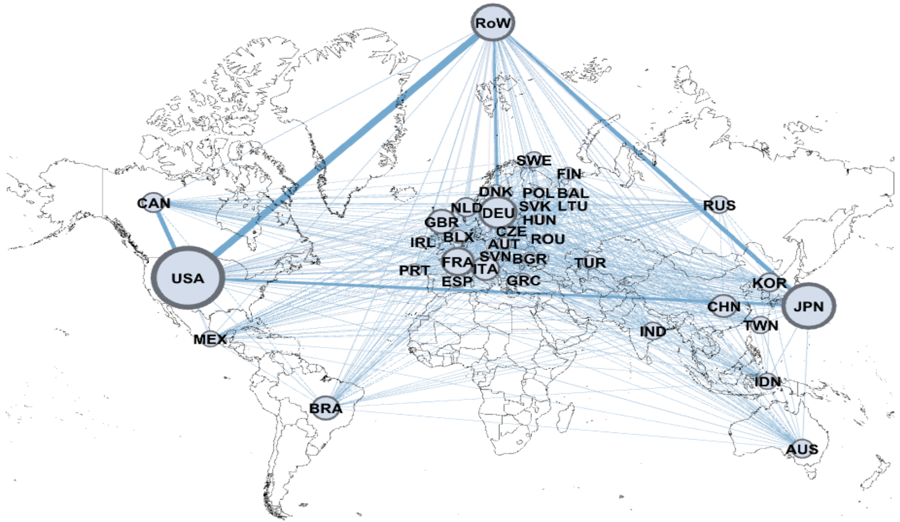
It is essentially a factor demand model, which means focusing on the **demand side** will help us enrich the gravity models.

Standard CES-gravity of aggregate trade⟶income shocks in a country generate proportional changes in national factor demands.

Substantial growth in world relative income⟶we need to ask whether or not traditional gravity mechanisms can be refined.

Are there other demand models that are more consistent with the data?

Global Demand for Foreign Factors in WIOD, 1995



- In the past, the CES-gravity models were constructed in such a way that it conforms with known empirical relationships. One way to test the model is to bring the theory of non-homotheticity—allowing income shocks to affect relative demand for imported factors.
- This paper introduces a different class of implicit consumer preferences (which nests a CES) into the gravity model.
- This paper estimates several nested demand systems (and test them using likelihood-ratio test and out-of-sample predictions) under gravity framework using the same data and estimation procedure.
- The devised estimation procedure allows identifying the elasticity of trade costs with respect to distance and border coefficients from the elasticity of trade flows with respect to trade costs.

Fajgelbaum and Khandelwal (2016)

- Almost-Ideal Demand System (AIDS)—n^{th} Order Approximation
- Similar to the translog expenditure system applied in Tan (2013), but adds non-homothetic characteristics.

Fieler (2011)

- Constant Relative Income Elasticity (CRIE)—Explicitly Direct

Bertoletti, Etro and Simonovska (2018)

- Indirectly Additive (IA) demand system—Explicitly Indirect

Comin, Lashkari and Mestieri (2015)

- Non-Homothetic CES (NHCES)—Implicitly Direct CES

Table: Demand Systems in Gravity Models

Gravity Literature	**Demand System**	**Parsimonious**	**Sub-Inc-Effect Separation**	**Non-CES**
Standard	CES	X		
Fajgelbaum *et al.* (2016)	AIDS			X
Fieler (2011)	Explicitly Direct	X		X
Bertoletti *et al.* (2018)	Explicit Indirect	X		X
Comin *et al.* (2015)	Implicitly Direct NHCES	X	X	
This paper	Implicitly Indirect CDE	X	X	X

"Parsimonious" $\approx$ number of parameters is proportional to n goods.

A utility function $v = u(x)$ is implicitly additive, if it may be defined by an identity of the form: $\sum_k F^i(x_i, v) \equiv 1$ (directly) or $\sum_i G^i(\frac{p_i}{E}, v) \equiv 1$ (indirectly). The value of u cannot be explicitly solved in terms of the model's exogenous variables.

Constant Difference of Elasticities (CDE) (Hanoch, 1975):
$G(\frac{p}{w}, u) = \sum_i \beta_i u^{e_i(1-\alpha_i)} (\frac{p_i}{E})^{1-\alpha_i} \equiv 1$, is implicitly indirectly additive.

The consumption behavior is governed by distribution parameter $\beta_i > 0$, expansion parameter $e_i > 0$ and substitution parameter $\alpha_i > 0$. The model is

- relatively parsimonious;
- separate substitution effects from income effects;
- non-CES, complementarities, inferior goods.

If $\alpha_i = \alpha \ \forall \ i$, then the CDE will collapse to an NHCES.

If $\alpha_i = \alpha \ \forall \ i$ and $e_i = 1 \ \forall \ i$, then the CDE will collapse to a standard CES.

The conceptual framework is similar to Anderson and van Wincoop (2003)/Armington. The estimation framework follows Balistreri and Hillberry (2006, 2007) (using MPEC).

With the derived CDE per capita demand (with $E_l = Y_l/L_l$), the nominal bilateral total trade flows is given by

$$X_{i,l} = \frac{\beta_i u_l^{e_i(1-\alpha_i)}(1-\alpha_i)(FOB_i\tau_{il})^{1-\alpha_i} Y_l^{\alpha_i} L_l^{1-\alpha_i}}{\sum_j \beta_j u_l^{e_j(1-\alpha_j)}(1-\alpha_j)(FOB_j\tau_{jl})^{1-\alpha_j} Y_l^{\alpha_j-1} L_l^{1-\alpha_j}}. \qquad (1)$$

Consumer observes the prices at origin i (FOB_i) adjusted by trade costs ($\tau_{il} = d_{il}^{\rho}[\exp(\delta_{il})]^{1-\text{dummy}_{il}}$) when trade units arrive at destination l.

The corresponding gravity equation is

$$\begin{aligned}\log X_{il} = {} & \log|\beta_i(1-\alpha_i)| + e_i(1-\alpha_i)\log u_l + (1-\alpha_i)\log FOB_i \\ & + \rho(1-\alpha_i)\log d_{il} + \alpha_i \log Y_l + (1-\alpha_i)\log L_l \\ & + (1-\text{dummy}_{il})(1-\alpha_i)\delta_{il} \\ & - \log\Bigg|\underbrace{\Bigg[\sum_j \beta_j u_l^{e_j(1-\alpha_j)}(1-\alpha_j)(FOB_j\tau_{jl})^{1-\alpha_j} Y_l^{\alpha_j-1} L_l^{1-\alpha_j}\Bigg]}_{\text{Multilateral Resistance}}\Bigg| + \varepsilon_{il},\end{aligned} \qquad (2)$$

Econometric Procedure

where the utility is jointly evaluated from

$$G(\frac{FOB}{E}, u) = \sum_i \beta_i u_l^{e_i(1-\alpha_i)} (\frac{FOB_i \tau_{i,l}}{E_l})^{1-\alpha_i} \equiv 1, \quad (3)$$

and the unit expenditure function under benchmark GE system.

Let us define the unit expenditure function as the true-cost-of-living index (price of utility $\neq$ average price index of goods details).

$$PU_l = \frac{\sum_i \beta_i u_l^{e_i(1-\alpha_i)-1}(1-\alpha_i)(FOB_i\tau_{il})^{1-\alpha_i} Y_l^{\alpha_i} L_l^{1-\alpha_i} e_i}{\sum_j \beta_j u_l^{e_j(1-\alpha_j)}(1-\alpha_j)(FOB_j\tau_{jl})^{1-\alpha_j} Y_l^{\alpha_j - 1} L_l^{2-\alpha_j}}. \quad (4)$$

It can be formulated as a complementarity problem. MCP1

Let K_{il}^0 denote the *distributed* endowment in i that is supplied to l discounted by iceberg melts, which equals the total quantity demanded in l from i: $K_{il}^0/\tau_{il} = Q_{il} = q_{il}L_l$. The total quantity supply from origin i to the world is thus given by $K_i^0 = \sum_l \tau_{il}Q_{il} = \sum_l \tau_{il}q_{il}L_l$, which equals the fixed total endowment at origin

$$K_i^0 = \sum_l \frac{\beta_i u_l^{e_i(1-\alpha_i)}(1-\alpha_i)FOB_i^{-\alpha_i}\tau_{i,l}^{1-\alpha_i}Y_l^{\alpha_i}L_l^{1-\alpha_i}}{\sum_j \beta_j u_l^{e_j(1-\alpha_j)}(1-\alpha_j)(FOB_i\tau_{i,l})^{1-\alpha_j}Y_l^{\alpha_j-1}L_l^{1-\alpha_j}}. \quad (5)$$

MCP2

K_i^0 can also be seen as the total quantity of output units that needs to be produced in order to meet the total demand in the globe. The product of it and prices yield the income definition in each region i

$$Y_i = FOB_i K_i^0, \quad (6)$$

while the benchmark utility equals benchmark per capita income adjusted by fitted benchmark equilibrium price index

$$u_i = \vartheta_i \frac{E_i}{PU_i}, \quad \text{where} \quad \vartheta_l = \sum_i e_i w_{il}^*. \quad (7)$$

Labor is the only factor and the labor employment is fixed in the model. Denote ϕ_i as Ricardian unit labor requirement, the market clearance in labor gives

$$L_i = \phi_i K_i^0. \quad (8)$$

MCP3

Division of (6) by (8) automatically implies zero profit condition for firms

$$\phi_i \frac{Y_i}{L_i} = \phi_i E_i = FOB_i. \quad (9)$$

MCP4

Following Gourieroux, Monfort and Trognon (1984), Silva and Tenreyro (2006) and from (2), the PPML estimator is chosen (to avoid zero trade flows) and is given by

$$
\begin{aligned}
X_{il} = \exp\Bigg\{ & \Big|\log \beta_i(1-\alpha_i)\Big| + e_i(1-\alpha_i)\log u_l + (1-\alpha_i)\log FOB_i \\
& + \rho(1-\alpha_i)\log d_{i,l} + \alpha_i \log Y_l + (1-\alpha_i)\log L_l \\
& + (1-\text{dummy}_{il})(1-\alpha_i)\delta_{il} \\
& - \log\Bigg[\Big|\sum_j \beta_j u_l^{e_j(1-\alpha_j)}(1-\alpha_j)(FOB_j\tau_{j,l})^{1-\alpha_j} Y_l^{\alpha_j-1} L_l^{1-\alpha_j}\Big|\Bigg]\Bigg\} + \Upsilon_{il} \\
= {} & \exp(x_{il}b_i) + \Upsilon_{il},
\end{aligned}
\tag{10}
$$

The constrained optimization problem is to maximize

$$
L(b_i) = constant - \sum_i \sum_l \exp(x_{il}b_i) + \sum_i \sum_l y_{il}x_{il}b_i, \tag{11}
$$

subject to equilibrium constraints (3)-(8) and CDE parametric restrictions.

Identification

	CDE				NHCES				CES			
PPML (w/o border charges)	1131.28				1097.86				1074.71			
PPML (with border charges)	1164.38				1163.43				1162.88			
ρ	0.269				0.129				0.113			
	α	β	e	δ	α	β	e	δ	α	β	e	δ
AUS	3.24	0.08	1.06	2.05	7.85	0.41	1.01	0.57	8.78	0.45	1.00	0.50
AUT	5.07	0.20	1.03	0.92		0.12	1.02	0.59		0.12		0.51
BGR	6.62	0.13	1.06	0.62		0.01	1.06	0.78		0.01		0.76
BRA	4.35	0.24	0.99	1.56		0.26	1.00	0.77		0.30		0.67
CAN	4.53	0.37	0.93	1.09		0.31	0.95	0.56		0.56		0.49
CHN	2.67	0.06	0.94	3.18		0.40	0.99	0.71		0.51		0.57
CZE	7.51	0.68	1.05	0.45		0.04	1.02	0.72		0.04		0.64
DEU	4.53	1.00*	1.03	0.79		1.00*	1.02	0.42		1.00*		0.36
DNK	4.57	0.11	1.05	1.07		0.10	1.03	0.58		0.10		0.49
ESP	6.32	1.77	1.01	0.65		0.24	1.00	0.65		0.27		0.58
FRA	4.58	0.63	1.03	1.02		0.58	1.02	0.55		0.59		0.48
GBR	3.95	0.30	1.01	1.08		0.53	1.02	0.44		0.55		0.38
IND	4.70	0.17	0.92	1.73		0.12	0.96	0.98		0.18		0.71
IRL	4.59	0.06	0.96	0.81		0.05	0.97	0.46		0.08		0.41
ITA	4.52	0.50	1.02	1.01		0.50	1.01	0.53		0.52		0.47
JPN	3.67	0.62	1.00	1.15		1.59	1.01	0.43		1.73		0.37
KOR	2.64	0.06	1.06	2.31		0.39	1.03	0.41		0.38		0.36
MEX	5.75	0.82	0.97	0.74		0.16	0.97	0.69		0.25		0.55
NLD	4.38	0.21	1.01	0.70		0.25	1.01	0.34		0.27		0.30
TWN	2.27	0.04	1.02	2.91		0.37	1.00	0.24		0.41		0.22
USA	4.77	5.39	1.00*	0.69		3.71	1.00*	0.40		4.20		0.36

*Normalized factors/countries: $\beta_{DEU} = e_{USA} = 1$, and $u_{JPN} = 10$.

- Due to implicit properties, the identification cannot be achieved using conventional methods (e.g., reduced-form regressions).
- The general equilibrium system is structurally estimated using a Mathematical Program with Equilibrium Constraints (MPEC), which has been applied to several gravity-trade literature (Balistreri and Hillberry, 2007; Balistreri, Hillberry and Rutherford, 2011; Tan, 2013).
- The non-linear program is evaluated in the General Algebraic Modeling System (GAMS) using the MPEC-NLPEC solver.
- The program tackles a solution to the implicit utility that is not of exact forms.
- It allows to estimate multiple nested demand systems using the same data (e.g., **1995** GDP, population, distance from **CEPPII** and **WIOD** trade flows) and underlying estimation procedure.
- Finally, the estimation procedure is devised such that the population variable helps the identification.

An Example of CES-Gravity

Thank You

- Unlike the standard CES, where $PU \equiv P$, the price index here carries an implied non-homothetic relationship with P

$$\frac{PU}{P} = \vartheta,$$

where $\vartheta = \sum_i e_i \omega_i$ is the expenditure-share weighted average of expansion parameters, satisfying

$$\omega_i \equiv \frac{p_i q_i}{E} = \frac{\beta_i (1-\alpha_i) u^{e_i(1-\alpha_i)} \left(\frac{p_i}{E}\right)^{1-\alpha_i}}{\sum_j \beta_j (1-\alpha_j) u^{e_j(1-\alpha_j)} \left(\frac{p_j}{E}\right)^{1-\alpha_j}}.$$

- We may also call ϑ the elasticity of price of goods with respect to the cost of utility. back

Inequality Constraints as an MCP1

The *production of utility* must be zero whenever strict inequality constraints hold in equilibrium

$$\frac{\sum_i \beta_i u_l^{e_i(1-\alpha_i)-1}(1-\alpha_i)(FOB_i\tau_{il})^{1-\alpha_i} Y_l^{\alpha_i} L_l^{1-\alpha_i} e_i}{\sum_j \beta_j u_l^{e_j(1-\alpha_j)}(1-\alpha_j)(FOB_j\tau_{jl})^{1-\alpha_j} Y_l^{\alpha_j-1} L_l^{2-\alpha_j}} \geq PU_l \quad \perp \quad U_l \quad \geq 0.$$

back

Inequality Constraints as MCP2-MCP4

The market clearing conditions imply that the strict equalities would hold if and only if the associated goods or factors are free of charge

$$K_i^0 \geq \sum_l \frac{\beta_i u_l^{e_i(1-\alpha_i)}(1-\alpha_i)FOB_i^{-\alpha_i}\tau_{il}^{1-\alpha_i}Y_l^{\alpha_i}L_l^{1-\alpha_i}}{\sum_j \beta_j u_l^{e_j(1-\alpha_j)}(1-\alpha_j)(FOB_i\tau_{il})^{1-\alpha_j}Y_l^{\alpha_j-1}L_l^{1-\alpha_j}} \quad \perp \quad p_{il} = FOB_i\tau_{il} \geq 0,$$

Similarly

$$L_i \geq \phi_i K_i^0 \quad \perp \quad PL_i = \phi_i \frac{Y_i}{L_i} \geq 0.$$

and

$$\phi_i \frac{Y_i}{L_i} \geq FOB_i \quad \forall i \quad \perp \quad K_i \geq 0,$$

back

CES Welfare Index and MCP in GE

- In the standard CES demand, it can be shown that the Hicksian welfare index is equivalent to the aggregate price of goods index, e.g., $PU \equiv P = (\sum_i \beta_i p_i^{1-\sigma})^{\frac{1}{1-\sigma}}$.
- So in the CES-gravity, the world can be represented by a Hicksian economy, satisfying $q_i = \frac{\beta_i p_i^{-\sigma}}{PU^{1-\sigma}} E$.
- Then in the GE gravity, this Hicksian welfare along can be characterized by a complementarity problem

$$\left(\sum_i \beta_i p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \geq PU \quad \perp \quad u \geq 0$$

$$PU = \frac{E}{u}$$

- This relationship can be implemented as a Mixed Complementary Problem (MCP) model in GAMS.

back