Estimation of an Implicit Additive Indirect Demand System

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23rd Annual Conference on Global Economic Analysis

June 2020

Background

- ▶ The empirical work on preference relationships that are more flexible than CES in applied general equilibrium models is limited.

- ▶ This is because econometric estimation of such demand systems has proven to be difficult.

Outcome

- ▶ We revisit theoretical demand literature from the 1960s and 1970s and bridge the gap between theoretical and empirical foundations of models that are implicit, additive and indirect.

- ▶ We demonstrate that direct maximum likelihood estimation of such demand systems is practical for the first time.

Introduction

▶ Computable general equilibrium (CGE) modelers prefer globally well-behaved relationships, but there are not many.

▶ Most of the models used in the CGE systems are limited to constant elasticity of substitution (CES) forms.

▶ CES is globally regular and parsimonious, but it implies unitary income elasticities and constant elasticities of substitution.

▶ While nesting CES functions is a partial solution, unitary income elasticities are still implied.

▶ An alternative approach is the implicit additive functional forms developed by HANOCH (1971, 1975).

Introduction (cont'd.)

▶ We revisit seminal papers on implicit, additive, globally well-behaved systems, e.g., HOUTHAKKER (1960), HANOCH (1971), and HANOCH (1975).

▶ These papers document the theoretical properties of demand systems that share these features.

▶ While these papers argue the importance of the flexibility and potential for reflecting empirical demand relationships, they have received limited attention for decades.

▶ One such model is the Constant Difference of Elasticities (CDE), an additive, implicit, indirect demand system.

▶ It is a candidate in the GTAP model for modeling private household demand.

The Demand Model

- ▶ A preference relationship is implicitly additive, if it may be defined by an identity of the form: $\sum_k F^k(x_k, u) \equiv 1$ (**direct case**) or $\sum_k G^k(p_k/w, v) \equiv 1$ (**indirect case**).

- ▶ The relationship is **implicit** because u cannot be explicitly isolated as a function of the model's variables and parameters.

- ▶ The model is **implicitly indirectly** specified as

$$G(p/w, u) = \sum_k \beta_k u^{e_k(1-\alpha_k)}(p_k/w)^{1-\alpha_k} \equiv 1.$$

- ▶ Consumption behavior is governed by distribution parameters $\beta_k > 0$, expansion parameters $e_k > 0$ and substitution parameters $0 \leq \alpha_k \leq 1$ or $\alpha_k > 1$ for all k.

Key Features: Parsimonious and General

Why choose CDE?

▶ Parsimonious: number of parameters being identified is linear in the number of goods.

▶ Implicit additivity makes price and income elasticities much more flexible.

▶ One can no longer get one set of elasticities from the other directly!

▶ Separates substitution effects from income effects because relationships between cross-price derivatives and income derivatives are eliminated.

Key Features: Relations to Other Systems

The CDE nests other well-known preference systems through parametric restrictions:

- ▶ The CDE allows complementarities between goods and/or inferior goods.

- ▶ If $\alpha_k = \alpha \; \forall \; k$, then the model collapses to a the non-homothetic CES.

- ▶ If $\alpha_k = \alpha \; \forall \; k$ *and* $e_k = 1 \; \forall \; k$, then with minor algebraic reorganization, it is shown to be a standard CES.

Estimation Challenges

▶ The models have generally been estimated for the production cases, see, e.g., HAWKINS (1977), MERRILEES (1982), DAR AND DASGUPTA (1985), SURRY (1993), HASHIMOTO AND HEATH (1995).

▶ YANG (2019) constructs a GE system of a gravity model of trade using the CDE preferences and structurally identifies key parameters using a mathematical algorithm with the help of data on geographic distances and regional population.

▶ The key challenge to estimation in the consumer preference context is due to non-observability of **utility** that is implicitly defined.

▶ Most approaches have involved approximations and/or transformations that thwart the identification of parameters.

Estimation Framework

▶ We follow a maximum likelihood estimation procedure for implicit additive direct models, e.g., CRANFIELD *et al.* (2002), PRECKEL, CRANFIELD AND HERTEL (2010), GOUEL AND GUIMBARD (2019).

▶ The log likelihood is the objective function with the determinate of the error covariance matrix.

▶ This is based on Cholesky factors of the cross-goods error covariance matrix to facilitate evaluation of the determinate.

▶ Since the constraints are highly nonlinear, take pains to ensure we have a feasible starting solution by first estimating the simpler forms (CES or NHCES) that are nested within the CDE, greatly facilitating computation.

Econometric Procedure (click here)

Estimation Framework (cont'd.)

- There is some ambiguity in the parameter restrictions in HANOCH (1971, 1975), and we show that these conditions can be slightly relaxed, facilitating estimation.
  Theorem 4.1 in Yang and Preckel (click here)

- CRANFIELD *et al.* (2002) explicitly estimated the cardinal value of utility for an implicit direct demand system.

- YANG (2019) estimated the cardinal utility from a gravitational CDE preferences of aggregate trade.

- Following these authors, we explicitly estimate the cardinal utility levels, and argue that those who estimate standard CES functions implicitly due the same.

Normalization Scheme

▶ Utility can only be identified up to a strictly increasing transformation, and we introduce a constant elasticity transformation of utility to demonstrate that parameters of the system are not identified. `Utility Transformation (click here)`

▶ This implies we can normalize the sum of the β's and e's resulting in a well-identified and well-scaled system, (i.e., $\sum_k \beta_k \equiv 1$ and $\sum_k e_k \equiv N$).

▶ COMIN, LASHKARI AND MESTIERI (2015) observe similar identification issues, employing a different normalization.

▶ Normalizing sums of these parameters avoids unfortunate choices of individual parameters to normalize, and resulting in distribution parameters satisfying $0 < \beta_k < 1$ and expansion parameters (exponents) e_k that are equal to unity on average.

Complied Data and Estimation Program

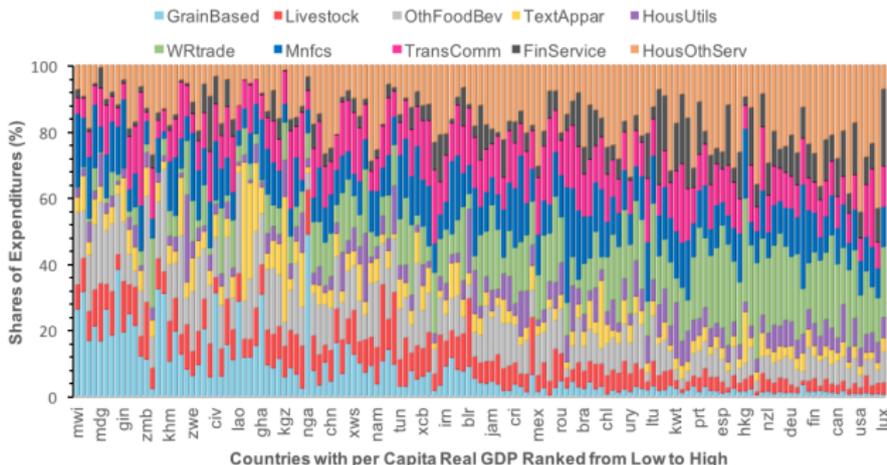▶ We use the aggregated GTAP version 9.2 and the World Bank's ICP databases with the reference year 2011.

Figure 1: Expenditure Shares across 141 Countries and 10 Aggregate Commodities.

▶ The estimation is formulated as a mathematical programming problem and solved in GAMS version 28.2.0 with CONOPT.

Estimation Results

Table 1: Estimated substitution, distribution and expansion parameters of the implicitly indirect function across 10 aggregate commodities.

Commodities	α's	β's	e's
GrainBased	0	4.950E-5	0.748
Livestock	0.567	0.002	0.588
OthFoodBev	0.858	0.056	0.710
TextAppar	0.931	0.321	1.189
HousUtils	0.886	0.055	0.482
WRtrade	0.754	0.104	1.383
Mnfcs	0.849	0.178	1.222
TransComm	0.871	0.118	0.745
FinService	0.819	0.144	1.565
HousOthServ	0.518	0.022	1.368

▶ To assess the **robustness** of parameter estimates, we developed a procedure to determine the **latitude for changing parameters**, finding it is in general on the order of a tenth of one percent.

▶ Parameter values cannot be changed substantially without reducing the likelihood value or violating constraints.

Estimation Results (cont'd)

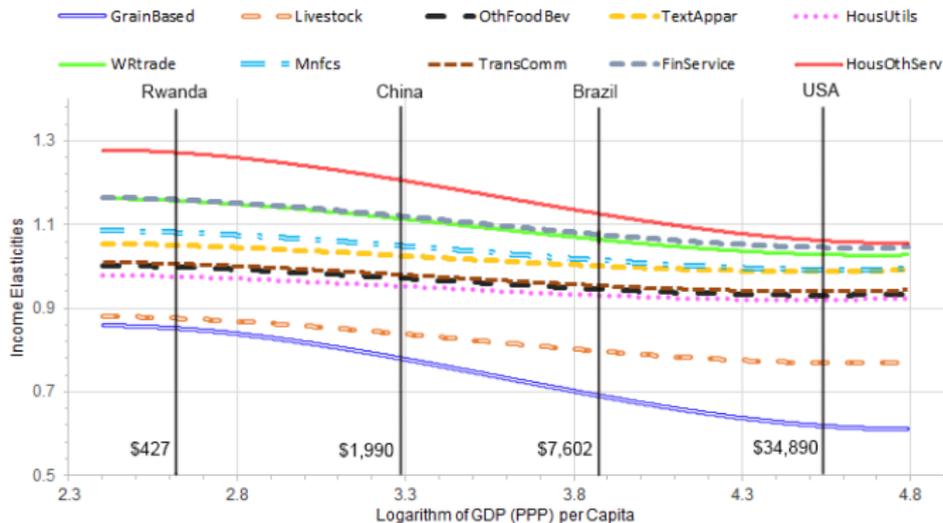▶ Our results show promising behavior of income elasticities:

Figure 2: Evolution of Income Elasticities with Real per Capita GDP (2011) (3rd Order Polynomial Trendline).

▶ We further develop a procedure of likelihood testing and parameter scales and find that the economic properties in terms of income and price elasticities of our estimated preference relationship are robust.

Conclusion

- ▶ We contribute to the literature by developing practical procedures for the estimation of an implict, additive, indirect preference system due to HANOCH (1971, 1975).

- ▶ In addition to the contribution of a formulation of the MLE problem, we demonstrate that there is no need for additional normalizations to identify parameters.

- ▶ A series of numerical tests indicate that the parameter values cannot be changed by an economically significant amount without reducing the likelihood function, suggesting that additional normalizations are not needed.

- ▶ Finally, we develop strategies for solution of this estimation problem using standard optimization software with reasonable computational expense.

Thank You!

Questions or comments?

Econometric Procedure

The log-likelihood function is given by

$$\log L = -0.5I[J(1 + \log 2\pi) + \log |\mathbf{R'R}|], \quad \mathbf{R} = [r_{km}]_{k,m=1,\dots,N}$$

where I and J are cardinal numbers of countries (or regions) and goods, respectively; $|\mathbf{R'R}|$ is the determinate of cross-goods error covariance matrix. As in GOUEL AND GUIMBARD (2019) with concerns of heterogeneity in the cross-country consumption patterns and potential heteroscedasticity, our added measurement errors (expressed in d's) are in quantities, instead of budget shares:

$$d_{ik} = q_{ik} - \widehat{q}_{ik} = \frac{w}{p_{ik}}(\Lambda_{ik} - \widehat{\Lambda}_{ik}), \quad \text{and}$$

$$\sum_n r_{nk}r_{nm} = \sum_k \frac{d_{ik}d_{im}}{I}, \quad r_{km} = 0 \ \forall \ m > k$$

Relaxed Global Regularity

Following Hanoch, we begin by setting up the cost minimization problem for the CRES subject to the defining constraint for the CRES relationship. This relationship has near-identical form to the CDE as follows:

$$\mathcal{H}(\mathbf{x}, u) = \sum_k \beta_k u^{-e_k(1-\alpha_k)}(x_k)^{1-\alpha_k} \equiv 1,$$

with the stated regularity conditions are: (i) $\beta_k, e_k > 0 \ \forall \ k \in N$, and (ii) either $\alpha_k > 1$ or $0 < \alpha_k \leq 1 \ \forall \ k \in N$.

Theorem
The latter set of regularity conditions in the CRES defining equation, i.e., condition (ii): either $\alpha_k > 1$ or $0 < \alpha_k \leq 1 \ \forall \ k \in N$, can be relaxed to either $\alpha_k > 1 \ \forall \ k \in N$, or $0 \leq \alpha_k \leq 1 \ \forall \ k \in N$, such that the formulated constrained optimization program (4.6) is still a convex program. see Proof.

Utility Transformation

Consider the transformation $u = \rho \upsilon^\delta$ where $\upsilon > 0$, which will be strictly increasing if $\rho > 0$ and $\delta > 0$. In this case, υ will be as good as u for explaining the data. Substituting the transformed u into the implicit utility defining equation:

$$
\begin{aligned}
1 \equiv G\left(\frac{\mathbf{p}}{w}, u\right) &= \sum_k \beta_k u^{e_k(1-\alpha_k)} \left(\frac{p_k}{w}\right)^{1-\alpha_k} \\
&= \sum_k \beta_k \left[\rho \upsilon^\delta\right]^{e_k(1-\alpha_k)} \left(\frac{p_k}{w}\right)^{1-\alpha_k} \\
&= \sum_k \beta_k \rho^{e_k(1-\alpha_k)} \upsilon^{\delta e_k(1-\alpha_k)} \left(\frac{p_k}{w}\right)^{1-\alpha_k} \\
&= \sum_k \widetilde{\beta}_k \upsilon^{\widetilde{e}_k(1-\alpha_k)} \left(\frac{p_k}{w}\right)^{1-\alpha_k},
\end{aligned}
$$

where $\widetilde{\beta}_k = \beta_k \rho^{e_k(1-\alpha_k)}$, and $\widetilde{e}_k = \delta e_k$.