Identification of Structural Parameters in the Hicksian Import Demand

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Exercises

- Map Hanoch's NHCES demand specification with Comin *et al.*
- Illustrate how the structural parameters (and trade costs) are identified in the gravity estimation.

Interested Structural Parameters

Demand parameters, distance elasticity, and border coefficients

CES Welfare Index and MCP in GE

- In the standard CES demand, it can be shown that the Hicksian welfare index is equivalent to the aggregate price of goods index, e.g., $PU \equiv P = (\sum_i \beta_i p_i^{1-\sigma})^{\frac{1}{1-\sigma}}$.
- So in the CES-gravity, the world can be represented by a Hicksian economy, satisfying $q_i = \frac{\beta_i p_i^{-\alpha}}{PU^{1-\alpha}} E$.
- Then in the GE gravity, this Hicksian welfare alone can be characterized by a complementarity problem

$$\left(\sum_i \beta_i p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \geq PU \quad \perp \quad u \geq 0 \tag{1}$$

$$PU = \frac{E}{u} \tag{2}$$

- This relationship can be implemented as a Mixed Complementary Problem (MCP) model in GAMS.

From Implicitly Indirect to Direct NHCES

- Now moving to a non-homothetic (NHCES) CES demand.
- The implicitly indirect NHCES demand function in Hanoch

$$G\left(\frac{\mathbf{p}}{w}, u\right) = \sum_i \beta_i u^{e_i(1-\alpha)} \left(\frac{p_i}{E}\right)^{1-\alpha} \equiv 1, \tag{3}$$

is identical to the implicitly direct NHCES

$$F(\mathbf{q}, u) = \sum_i k_i u^{e_i(g-1)} q_i^{1-g} \equiv 1, \tag{4}$$

if $\alpha = 1/g$ and $\beta_i = k_i^{\alpha}$.

Mapping to Comin *et al.*'s NHCES

Further, Equation (4) is identical to Comin *et al.*'s NHCES function

$$\sum_i (\Omega_i C^{\epsilon_i})^{\frac{1}{\sigma}} C_i^{\frac{\sigma-1}{\sigma}} \equiv 1, \tag{5}$$

if we impose

- $g = \frac{1}{\sigma}$, so $1 - g = \frac{\sigma-1}{\sigma}$ and $g - 1 = \frac{1-\sigma}{\sigma}$; implies $\alpha = \sigma$.
- $\beta_i = \Omega_i$, so $k_i = \beta_i^{\frac{1}{\sigma}} = \Omega_i^{\frac{1}{\sigma}}$.
- $\epsilon_i = e_i \frac{g-1}{g} = e_i(1-\alpha)$, so $e_i = \epsilon_i \frac{g}{g-1} = \frac{\epsilon_i}{1-\alpha}$.
- $Q \equiv u$ and $q_i = C_i$.

Hicksian Demand

- Under these parameter identities, both systems arrive at the same Hicksian demand:

$$q_i = \frac{\beta_i u^{e_i(1-\alpha)} p_i^{-\alpha} E}{\sum_i \beta_i u^{e_i(1-\alpha)} p_i^{1-\alpha}} = \beta_i u^{e_i(1-\alpha)} \left(\frac{p_i}{E}\right)^{-\alpha}, \tag{6}$$

that is equivalent to

$$C_i = \Omega_i \left(\frac{p_i}{E}\right)^{-\sigma} C^{\epsilon_i}. \tag{7}$$

- Let us show that we can derive a Hicksian welfare index from the implicitly indirect system

$$PU = \sum_i \beta_i e_i u^{e_i(1-\alpha)-1} \left(\frac{p_i}{E}\right)^{1-\alpha} E. \tag{8}$$

Hicksian Demand

- Unlike the standard CES, where $PU \equiv P$, the welfare index here carries an implied non-homothetic relationship with P

$$\frac{PU}{P} = \vartheta, \qquad (9)$$

where $\vartheta = \sum_i e_i \omega_i$ is the expenditure-share weighted average of expansion parameters, and $\omega_i = \beta_i u^{e_i(1-\alpha)} \left(\frac{p_i}{E}\right)^{1-\alpha}$, satisfying

$$P = \left[\sum_i (\beta_i p_i^{1-\alpha})^{\frac{1}{e_i}} (\omega_i E^{1-\alpha})^{\frac{e_i - 1}{e_i}} \right]^{\frac{1}{1-\alpha}}. \qquad (10)$$

- We may also call ϑ the elasticity of price of goods with respect to the cost of utility.

Hicksian NHCES Demand in MCP

- Similar to the CES-gravity, we can characterize the NHCES welfare index as an MCP, satisfying

$$\sum_i e_i \omega_i \frac{E}{u} \geq PU \quad \perp \quad u \geq 0 \tag{11}$$

$$\sum_i \omega_i \equiv 1, \tag{12}$$

where the demand is represented as

$$q_i = \beta_i \Big(\sum_i e_i \omega_i \Big)^{e_i(1-\alpha)} \left(\frac{E}{PU} \right)^{e_i(1-\alpha)} \left(\frac{p_i}{E} \right)^{-\alpha}. \tag{13}$$

- Note that (12) also implicitly characterizes the function of income $E \equiv \left(\sum_i \beta_i u^{e_i(1-\alpha)} p_i^{1-\alpha} \right)^{\frac{1}{1-\alpha}}$.

NHCES-Gravity Representation

- By adding appropriate subscripts i, l denoting exporting and importing countries and trade costs $\tau_{il} = d_{il}^{\rho}$, and letting FOB_i be the FOB price in the origin, while the population, $L_i = \frac{Y_i}{E_i}$, it can be shown that

$$\begin{aligned} \ln X_{il} = & \ln \beta_i + e_i(1-\alpha) \ln \Big(\sum_i e_i w_{il} \Big) \\ & + [e_i(1-\alpha) + \alpha] \ln Y_l + e_i(\alpha - 1) \ln PU_l \\ & + (e_i - 1)(1-\alpha) \ln L_l + (1-\alpha) \ln FOB_i \\ & + \rho_i(1-\alpha) \ln d_{il} + \varepsilon_{i,l}. \end{aligned} \tag{14}$$

- The Mathematical Programming with Equilibrium constraints (MPEC) solver will solve this model as a mixed problem of (Non-Linear Programming) NLP and MCP described earlier.

Structural Parameter Identification

Now we add the border coefficient δ_{il}, and modify the trade cost assumption as follows:

$$\tau_{il} = d_{il}^{\rho}[\exp(\delta_{il})]^{1-\mathrm{dummy}_{il}}, \tag{15}$$

where $\delta_{il} \equiv \ln(1 + \bar{T}_{il})$ and $\bar{T}_{il}$ is the tariff equivalent of border frictions; dummy_{il} is a dummy variable set of source-home consumption, with $\mathrm{dummy}_{il} = 0$ denoting cross-border shipments and $\mathrm{dummy}_{il} = 1$ if otherwise.

- With the welfare index definition, the MPEC will essentially unpack the log-linearised form of the gravity in (14).

Structural Parameter Identification

- The log-gravity as an empirical form essentially becomes

$$
\begin{aligned}
\ln X_{il} = {} & \ln \beta_i + e_i(1-\alpha)\ln u_l + (1-\alpha)\ln FOB_i \\
& + (1-\mathrm{dummy}_{il})(1-\alpha)\delta_{il} + \rho_i(1-\alpha)\ln d_{il} \\
& + \alpha \ln Y_l + (1-\alpha)\ln L_l + \varepsilon_{i,l}.
\end{aligned} \tag{16}
$$

- This indirect NHCES demand problem specified as a Hicksian-gravity is implicitly a bilevel program (BLP) involving so-called “leaders” and “followers”.

- The optimum value of utility and parameters in a given choice of objective functions (e.g., Least Squares, MLE, GMM, PPML) solves the optimum of marginal cost of utilities.

Structural Parameter Identification

- At fixed numeraire prices, the MPEC will then solve the shadow price of utility from the second-layer problem of "followers" as a constraint to the objective "leaders".
- In another way of saying, the MPEC essentially solves the marginal cost of utility such that $PU \equiv g(u)$ is a solution to a change in u with respect to variations in the model's observables in the objective function.
- This procedure allows us to control for u. Then when u is evaluated, we can identify the demand parameters, ρ and δ_{il}, given the data on income, population and distances.

Takeaway

- We essentially show the equivalence between Comin *et al.*'s direct NHCES function and Hanoch's indirect function.
- We then rationalize a welfare index from the indirect system, which is theory-consistent with Comin *et al.*
- This welfare representation can be characterized in a Hicksian economy of an MCP, similar to Balistreri and Hillberry (2006, 2007) and Balistreri, Hillberry and Rutherford (2011).
- The procedure allows evaluation of the utility index, which helps identifying structural demand parameters.