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NON-HOMOTHETIC PREFERENCES, OLIGOPSONY POWER AND RETURNS TO SCALE: A GENERAL EQUILIBRIUM ANALYSIS[1]

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What are the implications of non-homothetic consumer preferences on impacts of market power of downstream intermediaries on technological improvements in agriculture? What if there exists variable returns of scale on the production side at the same time? Do these more realistic environments of local economy mitigate the reduction in prices of intermediaries facing oligopsonistic agents who have market powers over local farmers? To answer these questions, this paper investigates the general equilibrium effects on oligopsony power by incorporating a model of non-homothetic demand which allows both subsistence and discretionary consumptions to vary with income and utility. We model a combination of decreasing return to scale upstream farmers and increasing return to scale downstream oligopsonistic processors, while allowing a Ricardian form of intersectorally mobile factor input requirement to reflect variations in sectoral comparative advantage in agriculture. Our analytical framework provides a different parametric channel to explain the general equilibrium effects on the endogenous market price mechanisms through idiosyncratic shifts of expenditure patterns, the degree of market power and the degree of returns to scale without relying on the traditional assumptions of constant returns to scale. We demonstrate that, when there is some confirmed degrees of oligopsony power represented by *conjectural variation*, the price of intermediate inputs is increasing in the returns to scale if the farmers' technology exhibits decreasing returns to scale. We find that, under competitive production equilibrium, upstream processors can be worse off due to a negative externality generated from an expansion of upstream farmers only if exogenous technical efficiencies and comparative advantages are homogenous across the two sides. It shows that, a technological improvement of upstream farmers will impact the *tangent* relationship between the price line and the production possibility frontier, hence the resource allocation among the two sectors. However, this result does not rely on the technology shifts of the upstream processors.

Keywords: Non-homothetic preferences, Oligopsony power, Returns to scale, General equilibrium effects.

1. INTRODUCTION

Recent published research incorporates general equilibrium (GE) perspectives to study direct and indirect spillover effects in local economies of developing countries (Filipski and Taylor (2012); Gupta *et al.* (2017); Taylor and Filipski (2014); Thome *et al.* (2013)). These papers extend a variety of important empirical program evaluation studies based on randomized experimental design or reduced form approaches (see, e.g., Angelucci and Giorgi (2009)). The GE models adopted in these papers offer a relatively more thorough and deeper view of the structural rural relationships within a local economy. However, the impacts of market power on spillover effects on income and the supply side of the local rural economies have been rarely studied in the past (Gupta (2019)).

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Gupta (2019) investigates the role of oligopsony power underlying a GE model of the local economy in Tanzania. With an integrated framework of GE effects of rural market structures and a classical model of market power, this paper finds that the oligopsony power of downstream cotton ginners significantly alters the spillover effects of a technological change in cotton production. The paper provides new insights of welfare gains of small farmers who can be affected by the market power. The consumer demand is modeled as a standard linear expenditure system (LES) originated in the Stone-Geary preferences, implying constant subsistence and discretionary budget shares. The final consumption is important as the author has a particular interest in the indirect spillover impacts on separate models of household groups who do not *work for* downstream oligopsony.

The impacts of market power of downstream farmers unambiguously enter the local economy through GE effects via the endogenous price mechanisms and quantity of intermediates. This is theoretically evident because the theory of *conjectural variation* tells us that the price of upstream farmers is partially determined by the degree of market power in the procurement of the farm supply (Gupta (2019)). However, GE models imply that an equilibrium impact is not only governed by farmers or producers but also by non-farm households. Therefore, under the more realistic context of non-homothetic consumption rather than the LES whose limit property is essentially homothetic, one important merit of this paper is showing that the benefits of technology improvement does not only depend on capacities of downstream farmers, market competitiveness, which are factors that belong to the production side, but also largely on the consumer preferences whose tastes and needs shifts in terms of variable subsistence and discretionary consumption across the spectrum of income levels based on Preckel, Cranfield and Hertel (2010)'s theoretical implicit indirect model.

Another contribution of this paper is to model variable returns to scales (RS) in farmer's production technology inspired by theoretical work in Panagariya (1981). We incorporate these technologies into the environment of imperfect competition which consists of upstream farmers and oligopsonistic upstream processors, while including a near-identical Ricardian form of factor input requirement to reflect variations in sectoral comparative advantage in agriculture. For the benefit of counterfactual exercises, we allow an exogenous technical efficiency shifter to interact with the industry's output and factor employment which jointly and parametrically determines farmer's output.

Our analytical framework provides a different parametric channel to explain the general equilibrium effects on the endogenous market prices through the degree of market power and the degree of returns to scale without relying on the traditional assumptions of constant returns to scale (CRS). We find that, when there is some confirmed degrees of oligopsony power, the price of intermediate inputs is increasing in the RS if the farmers are facing decreasing returns to scale (DRS) technology.

We further assume that the upstream farmers have DRS technology, while smaller sample of oligopsonistic upstream processors have IRS technology. We find that, under competitive production equilibrium, upstream processors can be worse off due to a negative externality generated from an expansion of upstream farmers only if exogenous technical efficiencies and comparative advantages are assumed to be homogenous across the two sides. This shows that, a technological improvement of upstream farmers will impact the *tangent* relationship between the price line and the production possibility frontier (PPF), hence the resource allocation among the two sectors. However, this result does not rely on the technology shifts of the upstream processors.

We connect the variable RS production framework back with the demand side. We develop a income definition supported by the GE theory to link with the concept of *supernumerary income* in an extended LES model in Preckel, Cranfield and Hertel (2010). We characterize the welfare representation of households in a *Hicksian* economy and develop a Hicksian consumer price index for this model for the purpose of general equilibrium analysis as implemented in GE literature of international trade such as Balistreri and Hillberry (2007) and Balistreri, Hillberry and Rutherford (2011).

The GE system developed in this paper, which involves multiple optimizers among upstream farmers, downstream processors and households, is formulated as a mathematical program with a mixed complementarity problem (MCP). While this type of problem is typically first manually formulated as a mathematical program with complementarity constraints (MPCC) using complementarity relationships and optimality conditions, and a conventional Mathematical Programming with Equilibrium Constraints (MPEC) can be a suitable algorithm (see, e.g., Balistreri, Hillberry and Rutherford (2011); Balistreri and Hillberry (2007); Tan (2012); Yang (2019)). We concerns that the solutions or sufficient solutions in the optimality conditions associated with the MCP are not guaranteed and that it is possible that the MPCC will only find local solutions but not global solutions. These concerns are stressed by Ferris *et al.* (2009) who develop an extended mathematical programming (EMP) annotations for chemical engineering applications. Given the recent software advancement, we solve our GE system using EMP annotations which generate first-order conditions and reformulate our problem as an MCP with solver JAMS as a reformulation tool.

2. MODEL SETUP

We adopt a GE framework that is conceptually similar to the GE local economy wide impacts evaluation (LEWIE) method demonstrated in Taylor and Filipski (2014) and applied in Gupta (2019) for examining the change in the effects of agricultural technological improvements due to market power of downstream intermediaries. In our model, we adopt a more flexible demand system to allow subsistence consumption and discretionary expenditures to vary with income. The model is essentially a generalization of the LES. On the production side, we allow non-constant RS production function as modeled in Panagariya (1981) rather than the commonly adopted constant return to scale (CRS) Cobb-

Douglas function such as in Gupta (2019). We incorporate a parameter that is near-identical to the Ricardian form of factor input requirement to reflect variations in sectoral comparative advantage in agriculture. We also allow an exogenous technical efficiency shifter to interact with the industry's output and factor employment that jointly and parametrically determines farmer's output.[1]

Finally, we consider an economic environment with downstream oligopsonistic processors. The upstream farmers face internal decreasing returns to scale (DRS) production technology (increasing returns to scale or IRS s possible for some sectors by parametric conditions) and sell farm products to oligopsony who process the intermediates with other inputs into processed commodity, which are then supplied to final consumers that are subject to MAIDADS with respect to choices including the processed commodities and other farm products harvested by those who do not interact with oligopsony.

2.1. *Consumer Demand*

2.1.1. *A Generalized Linear Expenditure System*

The common practice in GE-LEWIE Models consider consumption patterns that follow a path of Stone (1954)'s LES (Taylor and Filipski (2014)). The major advantage of LES is its parsimony and this feature alone is important for almost all GE models, due to significantly lowered computation burden (especially when connecting with the supply side), to an *easier mode* of parameter identification, and to less worrisome normalization scheme that is related to identification, which ensures that the solution is unique to scale. The LES was generalized by Rimmer and Powell (1996) as An Implicitly Directly Additive Demand System (AIDADS) who make the marginal budget share more flexible and allow it to vary with real total expenditure or income. Preckel, Cranfield and Hertel (2010) modified the fixed subsistence consumption in both LES and AIDADS and allow it to vary with both income and the cardinal level of utility that is implicitly defined in the model. This modified demand system is as known as a modified AIDADS (or MAIDADS). It has been studied by Gouel and Guimbard (2019) and has been applied in computable partial equilibrium models of global agricultural supply and demand such as in Yang, Gouel and Hertel (2018). The advantage of MAIDADS beyond depicting rich nonhomothetic preferences is its relative parsimony in terms of the number of model parameters compared to other nonhomothetic system approximated by n^{th}–degree Taylor polynomial such as Deaton and Muellbauer (1980)'s An almost ideal demand system or AIDS.

Let us first denote X_k the quantity consumption of commodity k. It can be splitted by some quantities of subsistence portion Θ_k and some quantities of discretionary portion Γ_k. The function of quantity consumption by a representative

[1]From Section 2 and beyond, it might be useful to think "farmer (s)" as "firm (s)" as in other economic literature.

consumer is given by

$$X_k = \Theta_k + \Gamma_k, \tag{2.1}$$

where

$$\Theta_k = \underbrace{\frac{\delta_k + \tau_i e^{\omega u}}{1 + e^{\omega u}}}_{\text{Subsistence Consumption}}, \tag{2.2}$$

and

$$\Gamma_k = \underbrace{\frac{\overbrace{\alpha_k + \beta_k e^u}^{\text{Marginal Budget Share}}}{1 + e^u} \frac{1}{p_k}\left(\mu - \sum_j p_j \underbrace{\frac{\delta_j + \tau_j e^{\omega u}}{1 + e^{\omega u}}}_{\text{Subsistence Consumption}}\right)}_{\text{Discretionary Consumption}}, \tag{2.3}$$

where k indexes commodities, respectively, all the Greek letters are the model parameters, p and μ denote price and income, respectively, and u is the implicit utility level. As per capita incomes (or the level of real total expenditure) rise, subsistence consumption converges asymptotically to τ and marginal budget share, in the same fashion, to β. They can increase or decrease with the level of per capita income, which is based on the signs of $\tau - \delta$ and $\beta - \alpha$. The discretionary consumption level, on the other hand, is governed by the marginal budget share, i.e., $(\alpha_k + \beta_k e^u)/(1 + e^u)$, but also by the discretionary income (i.e., income minus total expenditures on subsistence consumptions or *disposable* income). The speed parameter, $\omega \geq 0$, appeared in the expression of subsistence consumption, allows the speed of transition in the subsistence consumption to be different from the discretionary consumption. The model as it shows has $4n + 1$ parameters, but the identification requires that we effectively pin down $4n + 2$ parameters. This is because that the model is implicit rather than explicit, thus we need to simultaneously calculate the cardinal value of utility along with other parameters (Yang and Preckel (2020)).

In the LES, the level of subsistence quantity Θ_k is always held fixed, and the marginal budget share is some constant term Ω_k. This can happen in the MAIDADS system. To see this, if we allow $\alpha_k = \beta_k \ \forall\ k$ and $\delta_k = \tau_k \ \forall\ k$, then the MAIDADS effectively collapses to a standard LES system:

$$X_k = \Theta_k + \frac{\Omega_k}{p_k}\left(\mu - \sum_j p_j \Theta_j\right), \tag{2.4}$$

where $\Theta_k \equiv \delta_k = \tau_k$ and $\Omega_k \equiv \alpha_k = \beta_k$.

2.1.2. *Hicksian Consumer Price Index*

We solve an operational GE system in this paper using a Hicksian consumer welfare index that has been used in GE models of international trade. The computation methodologies to be discussed later involves deriving complementarity relationships under necessary and sufficient optimality conditions using Karush-Kuhn-Tucker Theorem (KKT). This requires that we solve a consumer price index from the MAIDADS system. It is commonly known in the GE and international trade literature that this price index equals the inverse of the Lagrange multiplier derived from the utility maximization problem (see, e.g., Dixit and Stiglitz (1977); Melitz (2003); Yang (2019)). Following Preckel, Cranfield and Hertel (2010), we can be obtain this index indirectly via the first-order conditions to the constrained maximization program:

$$\begin{aligned} \underset{u, X_k \forall k \in \{1,\ldots,N\}}{\text{maximize}} \quad & u \\ \text{subject to:} \quad & (1) \sum_k \frac{\alpha_k + \beta_k e^u}{1+e^u} \ln\left(X_k - \frac{\delta_k + \tau_k e^{\omega u}}{1+e^{\omega u}}\right) \\ & \quad - \ln(A) - u \equiv 1 \\ & (2) \sum_k p_k X_k \leq \mu \\ & (3)\ \ X_k > \frac{\delta_k + \tau_k e^{\omega u}}{1+e^{\omega u}}, \end{aligned} \tag{2.5}$$

where (1) in equation (2.5) is the implicit utility defining constraints, (2) the budget constraints and (3) is required to guarantee that (1) is defined so that supernumerary income net of discretionary consumption is positive. The inverse of Lagrange multiplier representing marginal income of utility is given by

$$\begin{aligned} PU = -\Bigg\{ & \sum_k \left[\frac{(\beta_k - \alpha_k)e^u}{(1+e^u)^2} \ln\left(X_k - \frac{\delta_k + \tau_k e^{\omega u}}{1+e^{\omega u}}\right) \right. \\ & \left. - \frac{\alpha_k + \beta_k e^u}{1+e^u}\left(X_k - \frac{\delta_k + \tau_k e^{\omega u}}{1+e^{\omega u}}\right)^{-1} \frac{(\delta_k - \tau_k)\omega e^{\omega u}}{(1+e^{\omega u})^2}\right] - 1 \Bigg\} \\ & \left(\mu - \sum_k p_k \frac{\delta_k + \tau_k e^{\omega u}}{1+e^{\omega} u}\right) u^{-1}, \end{aligned} \tag{2.6}$$

where PU is the defined consumer price index equaling the marginal income of utility. One might concern that equation (2.6) seems slightly lengthy thus about the correctness of it. The derivation, however, is straightforward. One way to gain some confidence is to check its transformations into other systems based on parametric conditions. If we let $\alpha_k = \beta_k \ \forall \ k$ and $\delta_k = \tau_k \ \forall \ k$, then PU

collapses to $(\mu - \sum_k \overline{\Gamma}_k)/u$, which is a familiar result that can be derived from the LES system. Furthermore, if we allow $\Gamma_k = 0$ along with $\alpha_k = \beta_k \ \forall \ k$ and $\delta_k = \tau_k \ \forall \ k$, this yields a Cobb-Douglas demand system, and PU collapses to μ/u, which is a general result for any homothetic demand system. We will discuss our computation and identification strategies using equation (2.6). Another way to verify the correctness of equation (2.6) is to take the total derivative of the first set of constraints (1) in its indirect form (e.g., Chen (2017) and Yang (2019)).

2.2. *Production Technology*

2.2.1. *Industry-Driven Returns to Scale*

We assume that under competitive equilibrium farmers in agricultural sectors encompass a homogenous production technology with DRS. Farmers f produce commodities k with the output X_{kf} employing labor L_{kf}. The total output of k produced in the economy is thus given by

$$X_k = \sum_f X_{kf}, \tag{2.7}$$

with total labor amount of labor employed in producing commodity k:

$$L_k = \sum_f L_{kf}. \tag{2.8}$$

where

$$L = \sum_k L_k \tag{2.9}$$

is the fixed total endowment of labor who is perfectly mobile across sectors. The farmers' DRS production function is modeled as

$$X_{kf} = \underbrace{Z_{kf}}_{\substack{\text{Internal} \\ \text{efficiency}}} \overbrace{A_k^{-1} X_k^{\sigma_k}}^{\substack{\text{External} \\ \text{efficiency}}} L_{kf} = \Phi_{kf}^{-1} L_{kf}, \tag{2.10}$$

where

(2.11)

$$X_k = \sum_f X_{kf} = A_k^{1/(1-\sigma_k)} \Big(\sum_f Z_{kf} L_{kf} \Big)^{1/(1-\sigma_k)} = A_k^{-\eta_k} \Big(\sum_f Z_{kf} L_{kf} \Big)^{\eta_k},$$

and Z_{kf} controls the technology-biased (internal) efficiency in k's production of farmer f's technology. Note that these formulations are similar to what has been demonstrated in Panagariya (1981) (i.e., equations (2.12) and (2.13)) except that (1) we let $A_k > 0$, which is near-identical to the Ricardian form of unit labor requirement, a sectoral (external to farmer) efficiency shifter, to interact with the industry's output and employment that jointly and parametrically determines farmers' output; (2) we allow an exogenous technology efficiency (Z_{kf}) change due to a specific technological shift in the hope that an improvement may occur while farmers nonetheless face an unwishful DRS schedule; and (3) all non-processed commodities are subject to DRS rather than some of them to an IRS. The RS parameter $\sigma_k < 0$ (thus the auxiliary parameter $\eta_k < 1$) guarantees the long-run inefficiencies as the scale of production increases. Also note that while we believe external economies or diseconomies of scale definitely exist in all sectors across the globe, we do not, however, make any assumptions upon (at this point), thus the implementation as in the production theory only reflects ratios of labor employed and output quantities produced. One might immediately heed that the convenience of this treatment is its simplicity in making alternative assumptions about the RS across sectors (governed by parametric conditions) when reasonably needed, especially with considerations of today's increasingly *modernized factors* employed by farmers in the agricultural sector and firms in other sectors. We incorporate this assumption in Section 3, where IRS is internal to the downstream oligopsonistic processors when $\sigma_k > 0$ or $\eta_k > 1$ for some k. Finally, if we allow $A_k = Z_{kf} = 1 \; \forall \; k, kf$, then Equations (2.10) and (2.11) collapse to the canonical form of RS production function in Panagariya (1981), with the farmers' and sector's output expressed respectively as

$$X_{kf} = X_k^{\sigma_k} L_{kf}, \tag{2.12}$$

and

$$X_k = \sum_f X_{kf} = L_{kf}^{1/(1-\sigma_k)} = L_k^{\eta_k}. \tag{2.13}$$

2.2.2. *Income Linkages*

Equation (2.13) can be somewhat viewed as a special case of a standard Cobb-Douglas function differed by the bundles of inputs and parametric conditions. Furthermore, Φ_{kf} is the *de facto* unit labor requirement for producing commodity k by farmer f, which parametrically controls the sectoral output jointly with the intersectorally mobile labor. This term is not only a notational convenience but also provides a linkage between labor rents and output prices. To see this, note that by assumption, the total income M generated in the economy is

$$M = \sum_k p_k X_k. \tag{2.14}$$

By rearranging equation (2.10), we obtain individual farmer f's clearance in factor requirement in producing commodity k

$$L_{kf} = \frac{A_k X_{kf}}{Z_{kf} X_k^{\sigma_k}} = \Phi_{kf} X_{kf}. \tag{2.15}$$

The equation above motivates a relationship between demand for labor and the internal and external efficiency shifters as mentioned above. An upward shift in the sectoral labor demand A_k increases the supply of labor to farmer f by per unit of the output for commodity k, whereas an increase in the internal technological improvement specific to k by f's production will reduce the labor supply. Furthermore, the economy-wide market-clearing for labor requires that

$$L = \sum_f \sum_k \Phi_{kf} X_{kf}. \tag{2.16}$$

Dividing equation (2.14) by (2.16) implies zero-profit condition for farmer:

$$\mu \equiv \frac{M}{L} = \frac{\sum_k p_k X_k}{\sum_f \sum_k \Phi_{kf} X_{kf}}, \tag{2.17}$$

where μ is the per capita income equaling the same expression in the MAIDADS demand formula in equation (2.3), which is observable from the data.

3. AN OLIGOPSONY MODEL AND PRODUCTION EQUILIBRIUM

We now assume imperfect competition and present an environment of oligopsony. There exists R identical upstream farmers of farming commodity **1** who are subject to DRS given by equation (2.10) while facing S downstream oligopsonistic processors who process **1** along with other inputs into commodity **2**, and subsequently supply to final consumers who are subject to the implicit expenditure function (2.3) that is strongly additive with respect to quantities of $k \in \{1, ..., n-1\}$.

3.1. *Production-Possibilities Frontier*

For simplicity, we assume that all farmers have identical technological-biased efficiencies or let them be exogenous (i.e., its impact of interest only matters in the counterfactuals). Without loss of generality, we can normalize $Z_{kf} \equiv 1 \; \forall \; k$ in all f's technology such that the system of sectoral output equations

$$\begin{cases} X_k = A_k^{-\eta_k} L_k^{\eta_k} \\ X_{k+1} = A_{k+1}^{-\eta_{k+1}} \left(L - A_k X_k^{1/\eta_k}\right)^{\eta_{k+1}} \\ X_{k+2} = A_{k+2}^{-\eta_{k+2}} \left(L - A_k X_k^{1/\eta_k} - A_{k+1} X_{k+1}^{1/\eta_{k+1}}\right)^{\eta_{k+2}} \\ \quad\quad\quad\cdot \\ \quad\quad\quad\cdot \\ \quad\quad\quad\cdot \\ X_m = A_m^{-\eta_m} \left(L - \sum_{k \neq m}^n A_k X_k^{1/\eta_k}\right)^{\eta_m} \end{cases} \tag{3.1}$$

yields the PPF for upstream farmers as well as those who do not commercially interact with downstream processors who procure intermediate input **1** supplied by upstream farmers

$$\begin{aligned} \frac{dX_m}{dX_k} &= -A_m^{-\eta_m} \frac{\eta_m}{\eta_k} \left(L - \sum_{k \neq m}^{n} A_k X_k^{1/\eta_k}\right)^{\eta_m - 1} X_k^{(1-\eta_k)/\eta_k} \\ &= -A_m^{-\eta_m} \frac{\eta_m}{\eta_k} L_m^{\eta_m - 1} X_k^{(1-\eta_k)/\eta_k} \\ &= -A_m^{-\eta_m} \frac{\eta_m}{\eta_k} \left(A_m X_m^{1/\eta_m}\right)^{\eta_m - 1} X_k^{(1-\eta_k)/\eta_k} \\ &= -A_m^{-1} \frac{\eta_m}{\eta_k} X_m^{\frac{\eta_m - 1}{\eta_m}} X_k^{\frac{1-\eta_k}{\eta_k}} \end{aligned} \tag{3.2}$$

3.2. *Production Equilibrium*

Under competitive equilibrium, the costs of lending the amount of labor L_k must equal the values of commodity outputs X_k, thus

$$W L_k = p_i X_k \quad \forall \;\; k, \tag{3.3}$$

where the labor wage $W \gg 0$ for each commodity k is cleared as follows:

$$\begin{aligned} W &= A_k^{-1} p_k X_k^{(\eta_k - 1)/\eta_k} \\ &= \quad\quad \cdot \\ &= \quad\quad \cdot \\ &= \quad\quad \cdot \\ &= A_m^{-1} p_m X_m^{(\eta_m - 1)/\eta_m}, \end{aligned} \tag{3.4}$$

and for each pair of k and m, it follows that

$$\begin{cases} A_k \frac{W}{p_k} = X_k^{(\eta_k - 1)/\eta_k} \\ A_m \frac{W}{p_m} = X_m^{(\eta_m - 1)/\eta_m}. \end{cases} \tag{3.5}$$

To calculate the production equilibrium of farmers, we may use the slope of the PPF and rearrange equation (3.2):

$$-A_m \frac{\eta_k}{\eta_m} \frac{dX_m}{dX_k} = X_m^{(\eta_m - 1)/\eta_m} X_k^{(1-\eta_k)/\eta_k}. \tag{3.6}$$

Substituting equation (3.5) into equation (3.6) makes the sectoral technical efficiency parameter A_m disappear, while the condition for the production equilibrium satisfies that

$$\frac{p_k}{p_m} = -A_k \frac{\eta_k}{\eta_m} \frac{dX_m}{dX_k}. \tag{3.7}$$

Equation (3.7) demonstrates that the relationship between the price line and its position relative to the shape of the PPF depends on the relative magnitude of the RS parameters and the sectoral efficiency shifter A_k. With A_k being held fixed, we obtain the same results as in Panagariya (1981) with a convention where X_m is on the Y-axis. For every $0 < \eta_k < 1 < \eta_m$, the budget line is strictly flatter than the frontier thus cuts the PPF from below. With the RS ratio being fixed, for $A_k > 1$ the PPF shifts southwest towards the origin and can be sliced by the price line from above. When $A_k = 1$, farmers in sector m will be worse off due to a negative externality generated from an expansion of a farm in sector k. This subtly shows that, under competitive production equilibrium, given some nonzero ratios of any pairs of sector k and m, an improvement in the technical efficiency along X-axis with farmers producing commodity k will impact the *tangent* relationship between the price line and the PPF hence the resource allocation between k and m, but is independent of technical efficiency of the sector m on the vertical Y-axis.

3.3. *Implications of the Returns to Scale*

Let $up \in \{1, ..., R\}$ and $dn \in \{R+1, ..., R+S\}$, each respectively denotes DRS upstream farmers and IRS downstream processors, be a subset of farmers $f \in \{1, ..., N = R+S\}$ who produces $k = \{\mathbf{1}, \mathbf{2}, ..., \mathbf{n}\}$, where $\mathbf{1}$ is the specific intermediate good supplied by upstream farmers up to downstream oligopsonistic processors dn who finalize product $\mathbf{2}$ which, in turn, is delivered to consumers whose indifference curve portrays quantity choices among $k = \{\mathbf{2}, ..., \mathbf{n}\}$. Each farmer is assumed to have the same production functional form:

$$X_{kf} = \Phi_{kf}^{-1} L_{kf}. \tag{3.8}$$

However, the technologies of upstream farmers and downstream processors are differentiated by RS-driven parametric conditions. That is, $\sigma_k < 0$ ($\eta_k < 1$) for some $k \in \{1, ..., n-1\}$ harvested by some $up \in \{1, ..., R\}$ and $\sigma_{\mathbf{2}} > 0$ ($\eta_k > 1$) for commodity **2** processed by some $dn \in \{R+1, ..., R+S\}$:

$$\begin{cases} X_{k,up} = \Phi_{k,up}^{-1} L_{k,up} & \sigma_k < 0 \text{ (DRS)} \\ X_{\mathbf{2},dn} = \Phi_{\mathbf{2},dn}^{-1} L_{\mathbf{2},dn} & \sigma_k > 0 \text{ (IRS)}. \end{cases} \tag{3.9}$$

We also require that S is sufficiently small and that $R \gg S$ to reflect non-competitive behavior in the input market characterized by the oligopsony power of downstream processors. Note that from equation (3.5), we can derive the short-run farm supply elasticity ϵ_k, given by

$$\frac{p_k}{X_k} \frac{\partial X_k}{\partial p_k} = \epsilon_k = \frac{\eta_k}{1 - \eta_k} = -\frac{1}{\sigma_k} \quad \forall \ k. \tag{3.10}$$

It turns out that the right-hand side of this farm supply elasticity in equation (3.10) is similar to those derived from constant return to scale (CRS) Cobb-Douglas function. However, the intuitions of these two are different. Since CRS Cobb-Douglas requires that the sum of the output elasticities of factors to be unity, thus this supply elasticity can never be negative. In the case of DRS dependent on industry-output, the elasticity is strictly positive for $\sigma_k < 0$ ($\eta_k < 1$). In the case of IRS, the farm supply elasticity of oligopsonistic processors becomes negative for $\sigma_{\mathbf{2}} > 0$ ($\eta_{\mathbf{2}} > 1$), suggesting that the downstream processors are inelastic and that a change in price of the processed commodities has a negative impact on the change in their quantities supplied of commodity **2**. The *conjecture* is that, since the oligopsonistic processors do not have the market power over the output market, thus a significant increase in price of processed commodities is not influenced by them, but is rather transmitted from and driven by higher costs incurred at the stage of farming intermediaries due to exogenous shocks in the economy. An exogenously driven increased opportunity costs of production k implicitly reduce the market power of profit-maximizing oligopsonistic processors, and with that, their profits are strictly decreasing due to upward driven procurement costs. As a result, they supply less of commodity **2**. It worth mentioning that, while our analysis centers around the oligopsony power over the input market of commodity **1**, the degree of supply elasticity of processors do not interact with the non-competitive input prices (see Section 3.4).

3.4. *Conjectural Elasticity and Implications of Returns to Scale*

The inverse market demand function of farmer f's output is given by

$$p_k = p_k(X_k). \tag{3.11}$$

We can write the total and marginal cost function of the representative processor, respectively, as

$$\big[f(\mathbf{C}) + p_{\mathbf{1}}\big]X_{\mathbf{2},dn} + F \quad \text{(Total cost)} \tag{3.12}$$

and

$$f(\mathbf{C}) + p_{\mathbf{1}} \quad \text{(Marginal cost)}, \tag{3.13}$$

where $f(\mathbf{C})$ is variable processing costs that are increasing with farming output $X_{\mathbf{2},dn}$; $p_{\mathbf{1}}$ the price of intermediate purchase from upstream farmers; and F an avoidable fixed cost assigned to downstream processors. The representative processor maximizes the profit function, which is given by

$$\begin{aligned} \underset{X_{\mathbf{2},dn}\forall dn\in\{R+1,\dots,R+S\}}{\text{maximize}} \quad & \pi_{dn} = \big[P - f(\mathbf{C}) - p_{\mathbf{1}}\big]X_{\mathbf{2},dn} - F \\ \text{subject to:} \quad & \text{(1) Some technology capacity constraints;} \\ & \text{(2) Some physical capacity constraints;} \\ & \text{(3) Parametric restrictions.} \end{aligned} \tag{3.14}$$

where P is the representative processor's sales price of **2** at output market's competitive equilibrium. We assume that one unit of commodity **1** harvested by upstream farmers yields one unit of commodity **2** processed by downstream processors, i.e., $X_{\mathbf{1},up} = X_{\mathbf{2},dn} = X$, then the first-order condition gives

$$\begin{aligned} \frac{\partial \pi_{dn}}{\partial X} &= P - f(\mathbf{C}) - \left[p_{\mathbf{1}} + X\left(\frac{\partial p_{\mathbf{1}}}{\partial X_{\mathbf{1}}}\right)\left(\frac{\partial X_{\mathbf{1}}}{\partial X}\right)\right] \\ &= P - f(\mathbf{C}) - p_{\mathbf{1}}\left[1 + \left(\frac{X_{\mathbf{1}}}{p_{\mathbf{1}}}\frac{\partial p_{\mathbf{1}}}{\partial X_{\mathbf{1}}}\right)\left(\frac{\partial X_{\mathbf{1}}}{\partial X}\frac{X}{X_{\mathbf{1}}}\right)\right] \\ &= 0. \end{aligned} \tag{3.15}$$

It follows that from equation (3.10), we have

$$\frac{X_{\mathbf{1}}}{p_{\mathbf{1}}}\frac{\partial p_{\mathbf{1}}}{\partial X_{\mathbf{1}}} = \frac{1}{\epsilon_{\mathbf{1}}}, \tag{3.16}$$

and finally we may isolate and explicitly solve the price of per unit intermediate inputs

$$p_{\mathbf{1}} = (1 + \frac{\theta_{\mathbf{2}}}{\epsilon_{\mathbf{1}}})^{-1}[P - f(\mathbf{C})] = (1 - \sigma_{\mathbf{1}}\theta_{\mathbf{2}})^{-1}[P - f(\mathbf{C})], \tag{3.17}$$

where $\epsilon_{\mathbf{1}}$ is farm supply elasticity of commodity **1** of upstream farmers; $\theta_{\mathbf{2}} = (\partial X_{\mathbf{1}}/\partial X)(X/X_{\mathbf{1}})$ the conjectural elasticity which measures the degree of market power of downstream processors who produce commodity **2**; it is the belief of a representative processor about how its competitors will react if it varies its output of commodity **2** by one unit. Notably, the price of intermediate inputs is also driven by the nature of returns to scale in sector **1**. With DRS, $\epsilon_{\mathbf{1}} = \eta_{\mathbf{1}}/(1 - \eta_{\mathbf{1}}) = -1/\sigma_{\mathbf{1}}$ is increasing in the RS parameter $\eta_{\mathbf{1}} < 1$, thus the price of intermediate inputs $p_{\mathbf{1}}$ is increasing in $\eta_{\mathbf{1}}$ as long as $\theta_{\mathbf{2}} > 0$, i.e., when there is some confirmed degrees of oligopsony power. While $\theta_{\mathbf{2}} = 1$ characterizes a monopsonistic market of input **1**, i.e., $S = 1$, the case of perfect competition where $\theta_{\mathbf{2}} = 0$ suggests that the market price of input **1** will not be affected by any degrees of RS.

4. GENERAL EQUILIBRIUM

The GE system developed in this paper, which involves multiple optimizers among upstream farmers, downstream processors and households, is formulated as a mathematical program with a mixed complementarity problem (MCP). While this type of problem is typically first manually formulated as a mathematical program with complementarity constraints (MPCC) using complementarity relationships and optimality conditions, and a conventional Mathematical Programming with Equilibrium Constraints (MPEC) can be a suitable algorithm (see e.g., Balistreri, Hillberry and Rutherford (2011); Balistreri and Hillberry (2007); Tan (2012); Yang (2019)). We concerns that the solutions or sufficient solutions in the optimality conditions associated with the MCP are not guaranteed and that it is possible that the MPCC will only find local solutions but not global solutions. These concerns are stressed by Ferris *et al.* (2009) who develop an extended mathematical programming (EMP) annotations for chemical engineering applications. Given the recent software advancement, we solve our GE system using EMP annotations which generate first-order conditions and reformulate our problem as an MCP with solver JAMS as a reformulation tool.

(to be completed)

5. CONCLUSION

This paper investigates the general equilibrium effects on oligopsony power by incorporating a model of non-homothetic demand which allows both subsistence and discretionary consumptions to vary with income and utility. We model a combination of decreasing return to scale upstream farmers and increasing return to scale downstream oligopsonistic processors, while allowing a Ricardian form of intersectorally mobile factor input requirement to reflect variations in sectoral comparative advantage in agriculture. Our analytical framework provides a different parametric channel to explain the general equilibrium effects on the endogenous market price mechanisms through idiosyncratic shifts of expenditure patterns, the degree of market power and the degree of returns to scale without relying on the traditional assumptions of constant returns to scale. We demonstrate that, when there is some confirmed degrees of oligopsony power represented by *conjectural variation*, the price of intermediate inputs is increasing in the returns to scale if the farmers' technology exhibits decreasing returns to scale. We find that, under competitive production equilibrium, upstream processors can be worse off due to a negative externality generated from an expansion of upstream farmers only if exogenous technical efficiencies and comparative advantages are homogenous across the two sides. It shows that, a technological improvement of upstream farmers will impact the *tangent* relationship between the price line and the production possibility frontier, hence the resource allocation among the two sectors. However, this result does not rely on the technology shifts of the upstream processors.

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